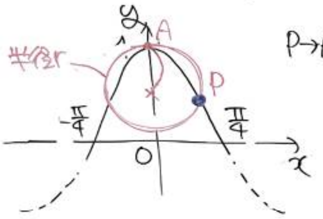
zemis2020-09-11（board）

$\boxed{487}$ $y=\cos 2x \left(-\frac{\pi}{4}\leqq x\leqq\frac{\pi}{4}\right)$



P→A

A(0,1)

P(p, cos2p) $\left(-\frac{\pi}{4}\leqq p\leqq\frac{\pi}{4},\ p\neq 0\right)$ とおく　P≠A

円 $x^2+(y-(1-r))^2=r^2$ にPが存在

$p^2+\{(\cos 2p-1)+r\}^2=r^2$

$p^2+(\cos 2p-1)^2+2r(\cos 2p-1)=0$

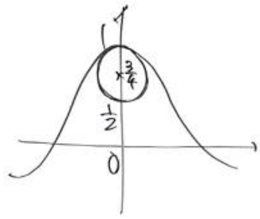
$r=\dfrac{p^2+(\cos 2p-1)^2}{-2(\cos 2p-1)}$

$=+\dfrac{p^2}{2(1-\cos 2p)}-\dfrac{1}{2}(\cos 2p-1)$

P→A のとき p→0 より

$\lim_{p\to 0} r=\lim_{p\to 0}\left\{\dfrac{1}{2}\times\dfrac{(2p)^2}{1-\cos 2p}\times\dfrac{1}{4}-\dfrac{1}{2}(\cos 2p-1)\right\}$　調節　1

$=\frac{1}{4}$ //

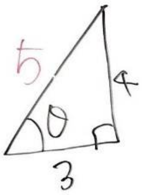


産医 2011

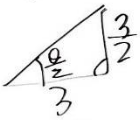
(1) $0°\leqq\theta\leqq 90°$. $\tan\theta=\frac{4}{3}$ $\left(\frac{\theta}{2}+\frac{\theta}{2}\right)$

⇨ $\tan\frac{\theta}{2}=\boxed{2}$

(14)



$4\times\frac{3}{3+5}=\frac{3}{2}$



$\tan\frac{\theta}{2}=\frac{1}{2}$ //

$\boxed{チャレ(61)}$ $a_n=\tan\frac{\pi}{2^{n+1}}$ $(n=1,2,3\cdots)$ $=2\theta$

(1) $a_{n+1}=\dfrac{1}{a_{n+1}}-\dfrac{2}{a_n}$ を証明.

$\left[a_{n+1}=\tan\frac{\pi}{2^{n+2}}\right]=\theta$ とおく

$\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$

$a_n=\dfrac{2\cdot a_{n+1}}{1-a_{n+1}^{\ 2}}$

$\dfrac{1}{a_n}=\dfrac{1}{2}\times\dfrac{1-a_{n+1}^{\ 2}}{a_{n+1}}$

$\dfrac{2}{a_n}=\dfrac{1}{a_{n+1}}-a_{n+1}$

$\therefore a_{n+1}=\dfrac{1}{a_{n+1}}-\dfrac{2}{a_n}$ ■

(2) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^n}\tan\frac{\pi}{2^{n+1}}$ を計算

$=\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{2^k}\cdot\tan\frac{\pi}{2^{k+1}}$ $=a_k$

部分和

$S_n=\displaystyle\sum_{k=1}^{n}\frac{1}{2^k}\tan\frac{\pi}{2^{k+1}}$ とおく

$=\displaystyle\sum_{k=1}^{n}\frac{a_k}{2^k}$ ← $\left(\text{(1)より } \cancel{\frac{2}{a_k}=\frac{1}{a_{k+1}}-a_{k+1}}\ ①\right)$

修正

$=\displaystyle\sum_{k=1}^{n}\frac{1}{2^k}\left(\frac{1}{a_k}-\frac{2}{a_{k-1}}\right)$　②で $n=k-1$

$=\displaystyle\sum_{k=1}^{n}\left(\frac{1}{2^k\cdot a_k}-\frac{2}{2^{k-1}\cdot a_{k-1}}\right)$　階差　k=1代入できない

$=\dfrac{a_1}{2^1}+\displaystyle\sum_{k=2}^{n}\frac{a_k}{2^k}$ ← (②で $n=k-1$ を代入)

$=\dfrac{a_1}{2}+\displaystyle\sum_{k=2}^{n}\frac{1}{2^k}\left(\frac{1}{a_k}-\frac{2}{a_{k-1}}\right)$

$=\dfrac{a_1}{2}+\displaystyle\sum_{k=2}^{n}\left(\frac{1}{2^k\cdot a_k}-\frac{1}{2^{k-1}\cdot a_{k-1}}\right)$

$=\dfrac{a_1}{2}+\left(\dfrac{1}{2^n\cdot a_n}-\dfrac{1}{2^1\cdot a_1}\right)$

$=\dfrac{1}{2^n\cdot\tan\frac{\pi}{2^{n+1}}}$ ← $\left(\varphi=\frac{\pi}{2^{n+1}}\text{ とおく}\right)$

$=\dfrac{2\varphi}{\pi}\cdot\dfrac{1}{\tan\varphi}=\dfrac{2}{\pi}\times\dfrac{\varphi}{\tan\varphi}$

$\left(a_1=\tan\frac{\pi}{2^2}=1\right)$

n→∞のとき φ→0 よ

$\therefore\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^n}\cdot\tan\frac{\pi}{2^{n+1}}$

$=\displaystyle\lim_{n\to\infty}S_n$

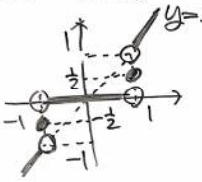
$=\dfrac{\pi}{2}$ //

62講

$\boxed{489}$ $\lim_{t\to 0}(1+t)^{\frac{1}{t}}=e \Leftrightarrow \lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=e$

(1) 1 (2) e^3 (3) e^{-4}

$\boxed{490}$ 復



$\boxed{491}$ $f(x)=\begin{cases} x+1 & (x\neq 1) \\ a & (x=1) \end{cases}$

$\therefore a=2$

平均値の定理

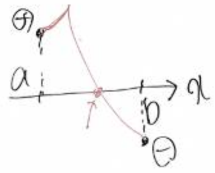
$\boxed{492}$ 中間値の定理 {p.270

$f(x)$: $\boxed{連続関数}$ ← 前提

$f(a)\times f(b)<0$ $(a<b)$

⟹ 方程式 $f(x)=0$ は

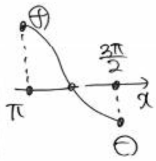
$a<x<b$ で少なくとも1つの実数解をもつ



$f(x)=\sin x - x\cdot\cos x$ とおく

$f(\pi)=\pi>0$

$f\left(\frac{3\pi}{2}\right)=-1<0$



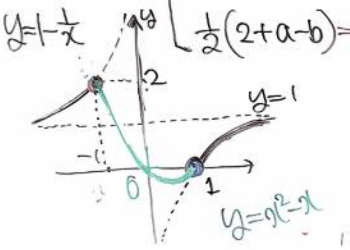
$f(x)=0$ は $\pi<x<\frac{3\pi}{2}$ で少なくとも1つの実数解をもつ

$\boxed{493}$ $f(x)=\lim_{n\to\infty}\dfrac{x^{2n}-x^{2n-1}+ax^2+bx}{x^{2n}+1}$ $\frac{\infty}{\infty}$とは限らない

$=\begin{cases} ax^2+bx & (|x|<1) \\ \frac{1}{2}(a+b)=0 & (x=1) \\ \frac{x-1}{x} & (|x|>1)\ \frac{\infty}{\infty} \\ \frac{1}{2}(2+a-b)=2 & (x=-1) \end{cases}$

$1-\frac{1}{x}$

$a=1, b=-1$



$\boxed{494}$ $\lim\frac{e^x-1}{x}=1$ $\left(\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}\right)$

(1) -5 (2) 2 (3) 2

(2) $\lim_{x\to 0}\dfrac{e^x-e^{-x}}{x}$

$=\lim_{x\to 0}\dfrac{e^x-1-(e^{-x}-1)}{x}$

$=\lim_{x\to 0}\left\{\dfrac{e^x-1}{x}-\dfrac{e^{-x}-1}{(-x)}\times(-1)\right\}$ 調整

$=1-1\times(-1)=2$

$\boxed{495}$ 略 代わりの問あり

(3) $\lim_{x\to 0}\dfrac{e^{x^2}-1}{1-\cos x}$

$=\lim_{x\to 0}\dfrac{e^{x^2}-1}{x^2}\times\dfrac{x^2}{1-\cos x}$

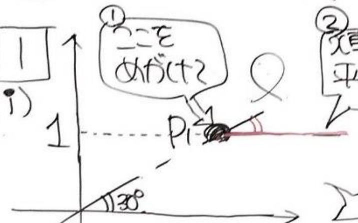
$=1\times 2$

$=2$

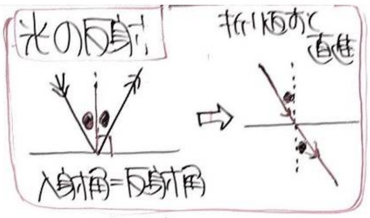
過去問めぐり 久留米

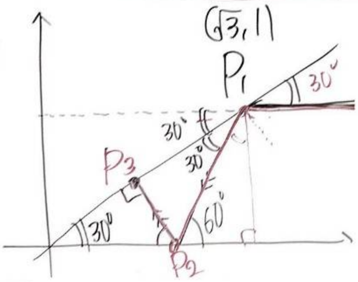
$\boxed{1}$

(i)



光の反射



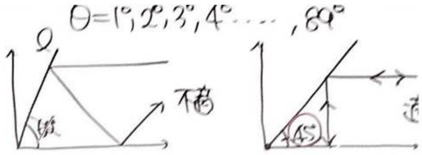


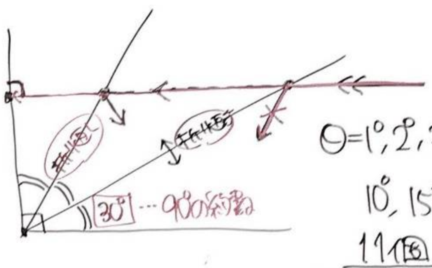
$n=\boxed{3}$ 具体化

$P_3\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

(ii)

$\theta=1°,2°,3°,4°,\cdots,89°$





$\theta=1°,2°,3°,5°,6°,9°,$
$10°,15°,18°,30°,45°$

11個

$\boxed{496}$ $n \geq 2$：自然数

$$x^{2n} = \overbrace{P_n(x)}^{\text{商}} \cdot \left(x^2 - x + \frac{n-1}{n^2}\right) + \overbrace{a_n x + b_n}^{\text{あまり}}$$

因数分解

$$= P_n(x)\left(x - \frac{1}{n}\right)\left(x - \frac{n-1}{n}\right) + a_n x + b_n$$

$x^2 - x + \frac{n-1}{n^2} = 0$

解の公式

$$x = \frac{1 \pm \sqrt{1 - 4 \cdot \frac{n-1}{n^2}}}{2}$$

$(\sqrt{\ })$ $\sqrt{\frac{n^2 - 4n + 4}{n^2}}$

$= \sqrt{\left(\frac{n-2}{n}\right)^2}$

$x = \frac{1}{n}$ 代入 $\left(\frac{1}{n}\right)^{2n} = \frac{1}{n} a_n + b_n$

$x = \frac{n-1}{n}$ 代入 $\left(1 - \frac{1}{n}\right)^{2n} = \left(1 - \frac{1}{n}\right) a_n + b_n$

$(1+0)^{\infty} \Rightarrow e$の定義

連立. $\left(1 - \frac{2}{n}\right) a_n = \left(1 - \frac{1}{n}\right)^{2n} - \left(\frac{1}{n}\right)^{2n}$

$$\therefore a_n = \frac{1}{1 - \underset{\to 0}{\frac{2}{n}}} \left\{ \underbrace{\left(1 - \frac{1}{n}\right)^{2n}}_{1^{\infty}} - \underbrace{\left(\frac{1}{n}\right)^{2n}}_{0^{\infty}} \right\}$$

よって $\lim_{n \to \infty} a_n = \underline{e^{-2}}$ // $\left[\left\{1 + \left(-\frac{1}{n}\right)\right\}^{-n}\right]^{-2} \to e^{-2}$

また $\lim_{n \to \infty} b_n = \lim_{n \to \infty} \left\{ \underbrace{\left(\frac{1}{n}\right)^{2n}}_{0^{\infty}} - \underbrace{\frac{1}{n}}_{\to 0} \times \underbrace{a_n}_{\to e^{-2}} \right\}$

$= \underline{0}$ //